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Fluxbranes: Moduli Stabilisation and Inflation

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ABSTRACT: Fluxbrane inflation is a stringy version of D -term inflation in which two fluxed D7-branes move towards each other until their (relative) gauge flux annihilates. Compared to brane-antibrane inflation, the leading-order inflationary potential of this scenario is much flatter. In the present paper we first discuss a new explicit moduli stabilisation procedure combining the F - and D -term scalar potentials: It is based on fluxed D7-branes in a geometry with two large four-cycles of hierarchically different volumes. Subsequently, we combine this moduli stabilisation with the fluxbrane inflation idea, demonstrating in particular that CMB data (including cosmic string constraints) can be explained within our setup of *hierarchical* large volume CY compactifications. We also indicate how the η -problem is expected to re-emerge through higher-order corrections and how it might be overcome by further refinements of our model. Finally, we explain why recently raised concerns about constant FI terms do not affect the consistent, string-derived variant of D -term inflation discussed in this paper.

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Contents

1	Introduction	1
2	Moduli Stabilisation - Basic Setup	5
2.1	Phenomenological Requirements	5
2.2	Moduli Stabilisation in the Large Volume Scenario - General Setup	6
2.3	A Two-Modulus Fluxbrane Inflation Model	8
3	Moduli Stabilisation - Hierarchical Setup	12
3.1	Cosmic Strings and the Need for a Hierarchy	13
3.2	String Loop Corrections	14
3.3	Moduli Stabilisation in a Modified Large Volume Scenario	15
4	Flat Directions for the Inflaton	18
5	Consistency of the Effective Theory	20
5.1	Issues in String D -Terms	20
5.2	Moduli Masses in Fluxbrane Moduli Stabilisation	21
6	Conclusions	23
A	Definitions and Conventions	24
B	F-Term Scalar Potential	25
C	Lower Bound on the Brane Flux Quanta	27

1 Introduction

Realising inflation in string theory has turned out to be a challenging problem. Amongst the models investigated in this context, many fall into a popular class known as brane inflation [1]. Here the inflaton is associated with the relative distance between a brane and an antibrane [2] or between two D-branes [3, 4].

This is a rather attractive setting because, in analogy to D -term [5, 6] or (more generally) hybrid inflation [7, 8], the energy density is dominated by a constant term as long as the branes are far apart. In this regime, the potential can be naturally flat. Later, once the branes have approached each other up to a certain critical distance, tachyon condensation takes the potential to zero almost instantaneously.

Unfortunately the simplest variant, where a brane and an antibrane annihilate at the end of inflation, cannot work since the size of the compact space does not allow for a

sufficient brane-antibrane separation [2]. In other words, in spite of the promising idea, the potential turns out to be too steep. While this can in principle be overcome by strong warping [9],¹ one is eventually forced to play various higher-order corrections off against each other [13–16]. This amounts to fine-tuning the inflaton potential such that inflation occurs close to an inflection point. One might thus feel that the original idea of ‘stringy hybrid inflation’ is lost.

To maintain the ‘hybrid inflation paradigm’, we therefore return to brane inflation models with two D-branes [3, 4]. The first of these is ‘inflation from branes at angles’ [3], which has its natural home in Type IIA orientifold models with D6-branes. Our fluxbrane proposal, to be discussed momentarily, can be viewed as the Type IIB mirror dual of this class of models. It hence has the enormous advantage of better control concerning moduli stabilisation. The second, D3/D7 inflation [4], has serious issues which we aim to overcome: In D3/D7 inflation, supersymmetry is broken by non-selfdual flux on the D7-brane. As the D7-brane is much heavier than the D3-brane it is, in general, the latter which moves in the geometry of the fluxed D7-brane. As it turns out, this realisation of D3/D7 inflation suffers from a similar problem as the brane-antibrane proposal [2], i.e. the potential is too steep. As possible ways out, a fine-tuned ‘small field’ version or the use of a highly anisotropic orbifold have been suggested in [17]. Alternatively, one might consider a setting where the fluxed D7-brane moves, probing the geometry produced by a large- N stack of D3-branes [18].

While these suggestions certainly warrant further detailed investigation, we believe that “fluxbrane inflation” [19] provides a more direct approach to ‘stringy D -term inflation’. The basic idea behind fluxbrane inflation is easy to state: The inflaton is associated with the relative distance of two D7-branes which carry non-selfdual worldvolume flux F . Once the branes come sufficiently close, (part of) the flux annihilates and the universe reheats. While this tachyon condensation process releases an energy density $\sim F^2$ (the constant term in the potential), the brane-distance-dependent attractive part of the potential is $\sim F^4$ (cf. section 1 of [19] for an intuitive explanation of this important technical fact). In other words, very schematically the inflaton potential has the form

$$V(\varphi) \sim F^2 + F^4 \log \varphi. \quad (1.1)$$

In the large-volume limit, where the flux density $F \rightarrow 0$, this gives us an obvious advantage over brane-antibrane scenarios: The φ -dependent term is more strongly suppressed than the constant, making the (leading order) potential flat enough for many e-foldings. Furthermore, when comparing to D3/D7 inflation (which is otherwise closely related), we do not have to appeal to the very large N of the D3-brane stack in order to make the D7-brane move. We finally note that, in addition to being T- or mirror-dual to [3], a different T-duality relates our setting to Wilson line inflation [20].

In the analysis of fluxbrane inflation in [19], moduli stabilisation was essentially taken for granted. This is a strong assumption for two reasons. On the one hand, our scenario

¹See, however, [10–12] for a recent discussion of potential problems with the supergravity solution corresponding to an $\overline{\text{D3}}$ -brane in a warped throat.

requires specific values for certain parameters of the compactification. It has therefore to be checked that these values can indeed be attained. On the other hand, the physical effects invoked to stabilise moduli tend to destroy the flatness of the inflationary potential, an effect well familiar also in other classes of brane inflation [9, 13, 21–23]. Hence, the flatness of the potential has to be checked *after* moduli stabilisation.

As we will demonstrate in some detail in the following, moduli stabilisation is possible in the phenomenologically required regime. This is the focus of the present analysis. Concerning the flatness of the potential after moduli stabilisation, we will at least be able to identify the most dangerous higher-order corrections. We will then present a strategy for suppressing them within our scenario, postponing a more thorough discussion to a further publication [24].

The size of curvature perturbations in fluxbrane inflation is governed by the inverse volume, forcing us into a regime where the volume \mathcal{V} is very large. Kähler moduli stabilisation is then naturally realised in the Large Volume Scenario [25, 26]. The latter is based on the interplay between α' - and non-perturbative corrections to Kähler and superpotential, resulting in a non-supersymmetric AdS vacuum. This vacuum is then uplifted by the D -term potential of fluxed D7-branes [27–34]. More specifically, as will be worked out in section 2, two independent flux-effects are required: One of them annihilates at the end of inflation, when the two relevant D7-branes meet. The other flux can not annihilate given that certain topological requirements are fulfilled. This flux is responsible for the uplifting to a Minkowski vacuum, which has to remain intact after reheating.

Appropriately suppressing the cosmic string contribution to CMB fluctuations is a crucial issue in brane or D -term inflation. In our scenario, the stability of cosmic strings is not completely trivial (cf. the discussion in [35]). To be on the safe side, we consider the worst-case scenario of topologically stable (local) cosmic strings rather than their semilocal cousins [35–39]. Cosmic string suppression then requires a hierarchy of four-cycle volumes in the internal manifold [19]. This forces us to go beyond the simple ‘warm-up’ model of section 2. Thus, in section 3, we embed our model of inflation in a *hierarchical* Large Volume Scenario, along the lines of [40, 41].

In this scenario, one is dealing with at least three Kähler moduli and stabilisation relies, in addition to α' - and instanton effects, on g_s - or loop-corrections to the Kähler potential [42–45]. We work out in detail how the three resulting contributions to the scalar potential combine with the D -term to give rise to a Minkowski/de Sitter minimum in which all Kähler moduli are stabilised.

To the extent that this is possible, we follow the discussion of the simple model of section 2. In particular, the Kähler modulus of the ‘small’ four-cycle carrying the instanton can be integrated out right away. We are then left with the Kähler moduli of the two large four-cycles. It is convenient to think in terms of the Calabi-Yau volume and a dimensionless ratio of two four-cycle volumes. The latter is fixed by the interplay of g_s -corrections and the D -term. Thus, one arrives at a fairly simple potential involving only the total volume. It is then possible to demonstrate stabilisation maintaining (almost) complete analytical control.

We end up with large overall volume ($\mathcal{V} \sim 3 \times 10^4$ in units of ℓ_s in the 10d Einstein

frame) and large tree-level superpotential ($W_0 \sim 4 \times 10^2$). The string coupling g_s is well in the perturbative regime ($g_s \sim 0.5$). Therefore, moduli stabilisation is achieved in a way consistent with all phenomenological requirements formulated in [19].

Such a large value of W_0 certainly bears the danger of violating the D3 tadpole constraint. In particular, translated into the language of F-theory compactifications, one needs an elliptically fibered fourfold X_4 with a large Euler characteristic $\chi(X_4)$ to allow for a sizable W_0 [46]. As we will see, the Euler numbers which have been found in [47] are sufficient for our purposes.

To have a convincing model of stringy D -term inflation, it is of course necessary to control inflaton mass corrections from the F -term potential. There are numerous sources for such terms. First of all, we have to choose the flux such that it does not stabilise the D7-brane position via an explicit appearance in the superpotential. The requirement for this (the flux has to be a two-form obtained by pullback from the ambient space) has already been discussed in [19]. Next, it is clear that the Kähler potential for D7-brane positions is non-minimal. More specifically, the D7-brane modulus ζ appears in the 4d supergravity Kähler potential in the form [48, 49]

$$K \supset -\log(S + \bar{S} - k(\zeta, \bar{\zeta})). \quad (1.2)$$

Here $k(\zeta, \bar{\zeta})$ is the Kähler potential on the moduli space of D7-brane positions and S is the axio-dilaton. It is thus apparent that, as soon as fluxes are turned on and some non-zero W_0 is generated, a mass term for ζ is induced in the F -term scalar potential. This generically leads to an η -problem even if W_0 is ζ -independent.

An exception occurs if k possesses a shift-symmetry, i.e. $k(\zeta, \bar{\zeta}) = k(\zeta + \bar{\zeta})$. This can be the case in $K3 \times K3$ and certain orbifold models (see [18, 23, 50, 51] for similar attempts in the context of D3/D7 inflation). Additionally, as we will explain in somewhat more detail in section 4, we expect that the moduli spaces of generic Calabi-Yau compactifications have corners where such a shift-symmetric structure arises at least approximately. However, this is not the end of the story: One can see from equation (1.2) that the moduli spaces of axio-dilaton S and D7-brane modulus ζ are intertwined in a non-trivial fibration. Hence, the S -dependence of W_0 potentially induces a non-trivial ζ -dependence.

Furthermore, g_s -corrections to the Kähler potential, which are used in section 3 to stabilise part of the Kähler moduli, are known to depend on open string moduli. All these effects can possibly spoil the nice properties of the fluxbrane inflation scenario. Section 4 is dedicated to investigating these issues. While we are optimistic that a viable region in the parameter space can be found, the analysis of this section is not yet conclusive. This set of problems is still under investigation [24].

Finally, we want to discuss our model in the light of an old question regarding the nature of the D -term in supergravity (see e.g. [35, 52]). Recently, this issue has received some renewed interest in the work of [53–59]. In particular, the authors of [53, 55] showed that in supergravities which emerge in the low-energy limit of a string compactification it is inconsistent to have a constant FI term. This is in complete agreement with the well-known structure of the D -term potential arising from string compactifications. These results also rule out models in which the FI term is dynamical at first, but then assumed

to be stabilised at a certain scale such that it can be viewed as a constant from the point of view of a supersymmetric low-energy effective theory. We address this issue in section 5 by computing the relevant moduli masses. It turns out that the forbidden regime with an effectively constant D -term can not be realised in our stringy setting and is indeed not necessary for D -term inflation to work.

2 Moduli Stabilisation - Basic Setup

2.1 Phenomenological Requirements

Fluxbrane inflation [19] is a stringy realisation of D -term hybrid inflation [5, 6]. In this scenario two D7-branes in a Type IIB orientifold compactification on a Calabi-Yau threefold X_3 are wrapped around two representatives of a holomorphic divisor class of X_3 (see [60] for a review). The modulus which describes the relative deformation of the two branes is associated with the 4d inflaton. Supersymmetry is broken by gauge flux on the D7-branes. To this end it is most convenient to describe the $U(1)$ gauge theories on the D7-branes in terms of their *overall* ($U(1)_+$) and *relative* ($U(1)_-$) piece. We will have more to say about gauge flux for the $U(1)_+$ theory later on. What is important for the inter-brane potential is the relative gauge flux \mathcal{F}_- in terms of which the effective potential for the canonically normalised inflaton φ is given by [19]

$$V(\varphi) = V_0 \left(1 + \alpha \log \left(\frac{\varphi}{\varphi_0} \right) \right) \quad (2.1)$$

with

$$V_0 = \frac{1}{2} g_{\text{YM}}^2 \xi_-^2, \quad \alpha = \frac{g_{\text{YM}}^4}{32\pi^3} \left(\int_{\text{D7}} J \wedge \mathcal{F}_- \right)^2, \quad \varphi_0^2 \sim \xi_- , \quad (2.2)$$

$$g_{\text{YM}}^2 = \frac{2\pi}{\frac{1}{2} \int_{\text{D7}} J \wedge J}, \quad \xi_- = \frac{1}{4\pi} \frac{\int_{\text{D7}} J \wedge \mathcal{F}_-}{\mathcal{V}}. \quad (2.3)$$

The D -term for $U(1)_-$ is denoted by ξ_- . In the above formulae it has already been implemented that the D3-brane charge induced by the relative flux vanishes, $\int_{\text{D7}} \mathcal{F}_- \wedge \mathcal{F}_- = 0$. As discussed in [19] this specialisation helps circumvent phenomenological constraints due to cosmic string production at the end of inflation. The reference field value in the logarithm is chosen to be the critical field value φ_0 below which a tachyon appears in the spectrum.² Throughout this paper we will measure 4d quantities in units of the reduced Planck mass M_p , while internal quantities such as lengths are measured in units of ℓ_s in the 10d Einstein frame (cf. appendix A).

As discussed in the introduction, the energy-density during inflation comes primarily from the constant term in the potential, while the logarithm presents only a small variation of that constant. When φ approaches the critical value φ_0 , tachyon condensation sets in and the universe reheats.

The potential (2.1) is subject to several phenomenological constraints [19]: First, one can show that the slow-roll conditions are satisfied very naturally in the large volume

²A different choice, $\varphi_0 \rightarrow \varphi_0 \times \text{const.}$, would be irrelevant at our level of precision.

regime. Secondly, the prediction for the spectral index n_s in terms of the number of e-foldings N

$$n_s \simeq 1 - \frac{1}{N} = 0.983 \quad \text{for } N = 60 \quad (2.4)$$

agrees with experiment at the level of 2σ ($n_s = 0.968 \pm 0.012$ at 1σ according to WMAP7 [61]). Finally, the amplitude of adiabatic curvature perturbations $\tilde{\zeta} \equiv V^{3/2}/V'$ is determined by measurements as

$$\frac{2N}{\tilde{\zeta}^2} = \frac{\alpha}{V_0} = \frac{2\mathcal{V}^2}{\frac{1}{2} \int_{D7} J \wedge J} \simeq 4.2 \times 10^8 \quad \text{for } N = 60. \quad (2.5)$$

Assuming that, for the present purposes, the internal manifold can be characterised by a single length scale R we find

$$R \simeq 11, \quad \mathcal{V} \simeq 1.7 \times 10^6. \quad (2.6)$$

To the best of our knowledge, the only way to obtain such a large volume in Type IIB string compactifications is in the Large Volume Scenario [25, 26]. In the remainder of this section we therefore outline how moduli stabilisation in fluxbrane inflation can work in principle in this setting.

2.2 Moduli Stabilisation in the Large Volume Scenario - General Setup

It was found in [25, 26] that under certain topological conditions there exists a non-supersymmetric AdS minimum of the scalar potential of Type IIB string theory compactified on a Calabi-Yau orientifold. This minimum appears at an exponentially large volume of the internal manifold and is therefore suitable for our purposes. To find this minimum one applies a two-step procedure: First, the complex structure moduli and the axio-dilaton are stabilised via bulk fluxes [62] and integrated out at a high scale, giving rise to a constant tree-level superpotential W_0 . Due to the ‘no-scale’ structure of the Kähler-moduli Kähler potential the resulting leading order F -term potential is identically zero. A non-zero scalar potential arises at subleading order through α' -corrections in the Kähler potential and non-perturbative corrections in the superpotential. In a second step one then minimises the effective potential for the Kähler moduli resulting from these higher order effects.

D3-instantons can wrap internal four-cycles of the Calabi-Yau manifold. The corrected superpotential is given by

$$W = W_0 + \sum_p A_p e^{-a_p T_p}, \quad (2.7)$$

where the $T_p = \tau_p + ib_p$ denote the complexified Kähler moduli of the instanton four-cycles. The constants a_p are given by $a_p = 2\pi$, while the Pfaffian prefactor A_p depends on the complex structure moduli and the axio-dilaton (which are assumed to be fixed already) as well as the open string moduli. The latter dependence could well be an issue for the viability of our brane inflation model. For example, it is known that in the presence of D3-branes the one-loop Pfaffian A_p involves the D3-brane position [13]. A similar effect was argued to occur for D7-branes which carry flux with non-vanishing induced D3-brane charge [63]. Recall from the discussion below (2.1) that our flux \mathcal{F}_- is chosen such that

the induced D3-brane charge vanishes (and the same can also be imposed on \mathcal{F}_+). The effect of [13, 63] is therefore not expected to occur in our setup. It remains an open question whether, for D7-branes, there is a possible open string dependence of the non-perturbative superpotential beyond these effects. In particular, one must sum over all flux configurations on the D3 instantons [64], which may introduce such a dependence via the flux induced D(-1) charge. This could be avoided in geometries for which $H^{(1,1)}$ of the instanton divisor only contains elements which are even under the orientifold involution, such that the instanton cannot carry flux [64].

The second ingredient apart from the superpotential (2.7) is the Kähler potential including α' -corrections (which can be shown to be the leading corrections in inverse powers of the volume [40])

$$K = -2 \log \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right) - \log(S + \bar{S}) + K_{\text{cs}}. \quad (2.8)$$

Here $\xi = -\frac{\zeta(3)\chi(X_3)}{2(2\pi)^3}$ and $\chi(X_3)$ is the Euler characteristic of the Calabi-Yau threefold X_3 . Thus the resulting α' -contribution to the potential is $\sim 1/\mathcal{V}^3$. On the other hand the non-perturbative corrections in the superpotential are exponentially small, leading to a contribution $\sim e^{-a_p \tau_p}/\mathcal{V}^2$. For both contributions to be equally relevant for creating a large volume minimum of the scalar potential, one has to require some of the four-cycles on which instantons are wrapped to be exponentially smaller than the overall volume of the threefold. Suppose that there is one such small four-cycle whose modulus we will call τ_s and whose intersection form is ‘diagonal’ with respect to all other four-cycles in the sense that the only non-vanishing triple intersection number involving τ_s is its triple self-intersection³ κ_{sss} . Then the volume can be written in terms of the four-cycle moduli as

$$\mathcal{V} = \tilde{\mathcal{V}}(\tau_{q,q \neq s}) - c\tau_s^{3/2}, \quad (2.9)$$

where c is related to κ_{sss} as $c = \frac{2^{3/2}}{3!\sqrt{\kappa_{sss}}}$. Furthermore, we require $\tau_{q \neq s} \gg \tau_s$ such that the overall volume \mathcal{V} is large, measured in units of ℓ_s in the 10d Einstein frame. In this limit the scalar potential is given by [25] (see also appendix B)

$$V_F(\mathcal{V}, \tau_s) = V_{0,F} \left(\frac{\alpha \sqrt{\tau_s} e^{-2a_s \tau_s}}{c\mathcal{V}} - \frac{\beta |W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{\xi \gamma |W_0|^2}{g_s^{3/2} \mathcal{V}^3} \right) \quad (2.10)$$

with α, β and γ some positive constants which depend only on $|A_s|$ and $V_{0,F}$ some overall g_s -dependent prefactor. Their precise form is given in (B.7). This potential is already minimised in the axionic (i.e. b_s) direction. Since it is only the absolute value of W_0 and A_s which enter (2.10), in the following we will write W_0 instead of $|W_0|$ etc.

Extremisation with respect to τ_s in the limit $a_s \tau_s \gg 1$ gives

$$a_s \tau_s = \log \left(\frac{2\alpha \mathcal{V}}{\beta c W_0} \right) - \frac{1}{2} \log \tau_s = \log \left(\frac{4a_s A_s}{3c} \frac{\mathcal{V}}{W_0} \right) - \frac{1}{2} \log \tau_s \quad (2.11)$$

³Note that in our conventions $\kappa_{sss} > 0$. As the small four-cycle with modulus τ_s is contractible to a point, this means that in the expansion of the Kähler form J the coefficient t_s of the (1,1)-form ω^s is negative, $t_s < 0$. Here, ω^s is Poincaré dual to the small four-cycle.

and thus

$$V_F(\mathcal{V}) \simeq V_{0,F} \left(\frac{\xi \gamma W_0^2}{g_s^{3/2} \mathcal{V}^3} - \frac{c \beta^2 W_0^2}{4 \alpha \mathcal{V}^3 a_s^{3/2}} \log^{3/2} \left(\frac{2 \alpha \mathcal{V}}{c \beta W_0} \right) \right), \quad (2.12)$$

where we have neglected terms $\sim \log \tau_s$. Both terms in the above expression roughly scale like $\sim W_0^2 \mathcal{V}^{-3}$. Generically, the same will be true for the value of the F -term potential at its minimum. As this minimum is AdS, we need some extra positive contribution to the energy density which lifts the potential at least to Minkowski. In our setup it seems most natural to do this via a D -term. As we will see momentarily, such a D -term scales like \mathcal{V}^{-2} and will thus, in general, give rise to a runaway potential for \mathcal{V} , unless the size of the above F -terms is enhanced. The latter enhancement can be achieved via a large W_0 . Considering only \mathcal{V} and W_0 , we expect from these arguments that roughly $W_0^2 \sim \mathcal{V}$.

Before addressing in detail this question of a dynamical runaway in the closed string moduli space, we first explain why potential instabilities in the open string moduli can generally be avoided geometrically: During inflation the uplifting D -term is due to both the relative gauge flux \mathcal{F}_- and the overall flux \mathcal{F}_+ on the two D7-branes. As detailed in [19], the end of inflation is marked by a generalised recombination process between the two D7-branes: \mathcal{F}_- is responsible for a tachyonic mode in the spectrum between both branes. The resulting condensation leads to a bound state between the two branes in which the relative $U(1)_-$ is higgsed. The remaining bound state continues to carry gauge flux \mathcal{F}_+ , whose D -term ξ_+ is responsible for the uplift to Minkowski/de Sitter after reheating. To guarantee stability of this D -term apart from the potential runaway in the Kähler moduli discussed below it must be ensured that no further condensation process occurs. The only such process would be a generalised recombination between the brane bound state and its orientifold image along their common locus or possibly a recombination between the bound state and a different brane stack in the model. The appearance of a tachyon depends on the pullback of \mathcal{F}_+ to the respective intersection loci and can thus be controlled by a suitable choice of flux, see [19] for details. In particular this requires an explicit choice of orientifold projection from which the brane-image brane intersection can be deduced. While we do not present such a concrete geometry in this work, these arguments are sufficient to show that a run-away in the open string sector is in general not a problem.

2.3 A Two-Modulus Fluxbrane Inflation Model

As an illustrative example consider a two-modulus swiss-cheese model similar to the one discussed e.g. in the original LVS publication [25]. In such a model the overall volume can be expressed in terms of the two four-cycle volumes τ_b and τ_s as

$$\mathcal{V} = b \tau_b^{3/2} - c \tau_s^{3/2} \quad (2.13)$$

where $b = \frac{2^{3/2}}{3! \sqrt{\kappa_{bbbb}}}$, $c = \frac{2^{3/2}}{3! \sqrt{\kappa_{ssss}}}$, and $\tau_b \gg \tau_s$. Wrapping the fluxed D7-branes around the large four-cycle D_b and choosing flux $\mathcal{F}_\pm = n_\pm [D_b]$ for the overall/relative $U(1)$ theory

$U(1)_\pm$ of the brane pair induces a D -term potential [19, 48, 49]⁴

$$V_D^\pm(\mathcal{V}) = \frac{1}{16\pi\mathcal{V}^2} \frac{(\int_{D7} J \wedge \mathcal{F}_\pm)^2}{\frac{1}{2} \int_{D7} J \wedge J} = \frac{1}{16\pi\mathcal{V}^2} 2n_\pm^2 \kappa_{bbb}. \quad (2.14)$$

The full scalar potential thus reads

$$V(\mathcal{V}) \simeq \frac{V_{0,F}}{\mathcal{V}^3} \left(\frac{\xi \gamma W_0^2}{g_s^{3/2}} - \frac{c\beta^2 W_0^2}{4\alpha a_s^{3/2}} \log^{3/2} \left(\frac{2\alpha\mathcal{V}}{c\beta W_0} \right) + \frac{2n^2 \kappa_{bbb}}{g_s} \mathcal{V} \right) \quad (2.15)$$

where $n^2 = n_+^2 + n_-^2$. Note that from now on we work in a gauge where $e^{K_{cs}} = 1$. Let $f(\mathcal{V})$ denote the term in the brackets on the right hand side of (2.15). Then, in the Minkowski minimum after annihilation of \mathcal{F}_- ($V_D(\mathcal{V}) = V_D^+(\mathcal{V})$) we have $f(\mathcal{V}_{\min.}) = f'(\mathcal{V}_{\min.}) = 0$. Vanishing of $f(\mathcal{V})$ in the minimum yields (to leading order in $a_s\tau_s \simeq \log \left(\frac{4a_s A_s}{3c} \frac{\mathcal{V}_{\min.}}{W_0} \right)$, using also $f'(\mathcal{V}_{\min.}) = 0$)

$$\frac{a_s}{g_s} \left(\frac{\xi}{2c} \right)^{2/3} = \log \left(\frac{4a_s A_s}{3c} \frac{\mathcal{V}_{\min.}}{W_0} \right) \quad (2.16)$$

which can be used to rewrite $f'(\mathcal{V}_{\min.}) = 0$ as

$$W_0 = \frac{2}{3} \frac{n_+^2 \kappa_{bbb}}{A_s g_s} \frac{e^{a_s \tau_s}}{\tau_s^{1/2}}. \quad (2.17)$$

Plugging this back into (2.16) gives

$$\mathcal{V}_{\min.} = \frac{c\kappa_{bbb}}{2a_s A_s^2} \frac{n_+^2}{g_s} \frac{e^{2a_s \tau_s}}{\tau_s^{1/2}}. \quad (2.18)$$

Setting $n_+ = 5$,⁵ $\kappa_{bbb} = 5$, $\kappa_{sss} = 1$ (such that $c = \sqrt{2}/3$),⁶ and $A_s = 1$ we find that for $\mathcal{V}_{\min.} = 1.7 \times 10^6$ the parameters can be chosen to lie in the phenomenologically viable regime (see figure 1): A value $\xi = 0.1$ implies $g_s = 0.25$. In view of equation (2.17) this means

$$W_0 = 1 \times 10^5. \quad (2.19)$$

It turns out that there is a tension between such a large W_0 and the requirement to cancel

⁴Since the flux of the relative $U(1)$ theory will annihilate upon brane recombination, we cannot use it for uplifting the minimum value of the potential to zero. Instead, we use V_D^+ for the Minkowski uplift, while V_D^- is some additional energy density which is present during inflation and which decays into standard model d.o.f. upon reheating.

⁵It turns out that there is a lower bound on n_+ which is easy to understand: As n_- is integrally quantised, for a given n_+ the uplift to de Sitter cannot be arbitrarily small. However, a large extra D -term from the relative $U(1)$ on top of the uplift to Minkowski may potentially wash out the de Sitter minimum for the volume modulus. Therefore, the relative change in the size of the D -term before and after inflation cannot be too large or, in other words, n_+ has a lower bound. An approximation and a numerical calculation of this lower bound is done in the phenomenologically interesting case discussed in section 3. In the present setup, a similar calculation would give $n_+ \geq 4$. For what follows we use a slightly more conservative $n_+ = 5$. It is then possible to show numerically (in analogy to section 3.3) that one can indeed obtain a Minkowski minimum for $n_- = 0$ which is uplifted to a stable de Sitter minimum for $n_- = 1$.

⁶Large intersection numbers tend to exacerbate the problems discussed below. Here and in the next section we take $\kappa_{bbb} = 5$ which is, for example, the triple self-intersection number of the quintic [65] and appropriate blow-ups thereof [66, 67].

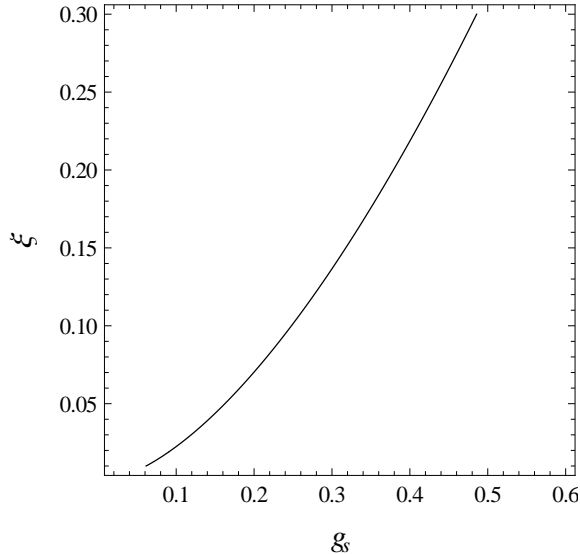


Figure 1. Allowed values for g_s and ξ in the simple two-Kähler moduli model.

the D3 tadpole: The authors of [46] were able to reformulate the tadpole cancellation condition in a way which makes it obvious that, as long as all F -terms for the complex structure moduli vanish in the minimum, W_0 or rather $\sqrt{g_s/2} W_0$ is bounded by $\sqrt{\chi(X_4)/24}$, where $\chi(X_4)$ is the Euler characteristic of the associated F-theory fourfold X_4 . In our example, $\sqrt{g_s/2} W_0 = 3.6 \times 10^4$, which would require $\chi(X_4) = 3.2 \times 10^{10}$. To the best of our knowledge, no fourfold with such a large Euler characteristic is known. Therefore, even before considering the production of cosmic strings, the simple two-Kähler moduli model turns out not to work quite generically. One can go ahead and try to choose manifolds with different intersection numbers and tune A_s etc. However, we will not go down this road because, to be on the safe side concerning the cosmic string bound referred to in the introduction, we have to consider models beyond this simple one anyhow (see the discussion in section 3.1). Instead, we will show that in a slightly more complicated situation W_0 can actually be much smaller, such that the tension described above is absent.

Some further comments are in order, which also apply to the more general setup discussed in section 3:

- As τ_s is given by equation (2.11) we find that, in view of (2.16),

$$\tau_s = \frac{1}{g_s} \left(\frac{\xi}{2c} \right)^{2/3}. \quad (2.20)$$

This can also be found using a different method (cf. appendix B).

- Uplifting an AdS vacuum through magnetised D7-branes has been discussed in different variants in [27–34]. Unless one appeals to a partial cancellation using charged fields [31–34], the D -term potential scales as $1/\mathcal{V}^2$. Since the F -term potential scales

as W_0^2/\mathcal{V}^3 , a successful uplift generically requires $W_0^2 \sim \mathcal{V}$. This is problematic for the following reason:⁷ Estimating the Kaluza-Klein scale on the basis of T^6 with equal radii, we have $m_{\text{KK}} = \sqrt{\pi}/\mathcal{V}^{2/3} \sim 1/\mathcal{V}^{2/3}$. At the same time $m_{3/2} \sim W_0/\mathcal{V}$, which should be parametrically smaller to justify the use of a 4d supergravity analysis. However, one finds (with W_0 normalised as in [62])

$$\frac{m_{3/2}}{m_{\text{KK}}} = \frac{\sqrt{g_s}}{4\pi} \cdot \frac{W_0}{\mathcal{V}^{1/3}} \sim \mathcal{V}^{1/6}. \quad (2.21)$$

This ‘goes the wrong way’ at large \mathcal{V} (though it does so very weakly). In [31] it was argued that due to the appearance of a large numerical factor $16\pi^4$ in the denominator of V_D it is possible, for $\mathcal{V} \sim 10^3$, to uplift the AdS minimum to a stable de Sitter vacuum with $W_0 = \mathcal{O}(1)$. In view of (2.15) we believe that the situation is not quite as simple: The only relative factors of 2π between F - and D -term contributions come from the definition of ξ . They suppress the F -term, making the situation naively worse, but can be easily compensated by a large $\chi(X_3)$. However, using the explicit formulae (2.17), (2.18) and (2.20), we can make equation (2.21) more precise:

$$\frac{m_{3/2}}{m_{\text{KK}}} = \frac{n_+}{3\sqrt{\pi}} \sqrt{\frac{\kappa_{bbb}}{c\sqrt{\tau_s}}} \cdot \mathcal{V}^{1/6}. \quad (2.22)$$

Assuming $n_+ \sim c \sim \kappa_{bbb} \sim \tau_s \sim 1$ this suggests that, at least in rough numerical agreement with [31], a fairly large \mathcal{V} can indeed be tolerated in spite of the ‘parametrical’ clash between $m_{3/2}$ and m_{KK} . However, it is not clear that a manifold of swiss-cheese type with such intersection numbers exists. Furthermore, as elucidated above, $n_+ = 1$ does not allow for a stable de Sitter uplift. Larger intersection numbers and a larger value of n_+ both deteriorate the situation, reducing the maximal size of the overall volume consistent with the requirement $m_{3/2} < m_{\text{KK}}$. On the other hand, the four-cycle volume τ_s , which could in principle suppress the size of the ratio (2.22), is essentially fixed at a value ~ 1 by (2.18) and the requirement $\mathcal{V}_{\text{min.}} = 1.7 \times 10^6$. In particular, with the numbers used and computed in this section we find $m_{3/2}/m_{\text{KK}} \simeq 34$ which means that there is no regime in which the supergravity approximation is valid.

The authors of [31, 68] furthermore propose to use warping to suppress the D -term even further. While this is certainly a very appealing possibility, we are hesitant to include it in our scenario: We fear that it might clash with the shift-symmetry that we need to keep our inflaton potential flat.

In fact, as we will see in a moment, our suggested solution to the ‘ D -term-suppression problem’ is a moderate hierarchy: Given that we have two large 4-cycles with significantly different volumes, we can arrange for the D -term to be parametrically smaller than $1/\mathcal{V}^2$.

In other proposals [31–34] the stabilisation mechanism crucially depends on the presence of non-trivial vevs for some of the charged matter fields which appear in the

⁷ We thank Joseph Conlon and Fernando Quevedo for pointing this out.

D -term. These would arise from the intersection of the mobile D7-brane with other branes in the compactification. A suitable choice of gauge fluxes can in general ensure the absence of such matter fields. Indeed this conforms with our assumptions described at the end of section 2.2 concerning absence of extra instabilities in the open string sector.

Finally, in [29] the authors consider only one Kähler modulus which is charged under the anomalous $U(1)$ and which also appears in the non-perturbative superpotential.

Other proposals for uplifting mechanisms put forward in the recent literature include [68, 69].

- From $f'(\mathcal{V}) = 0$ we find that the D -term contribution to the scalar potential (i.e. the third term in (2.15)) is suppressed by a factor of $a_s \tau_s = \frac{a_s}{g_s} \left(\frac{\xi}{2c}\right)^{2/3}$ relative to the first and second term in that expression. This means that the required uplift (i.e. the value of the F -term potential at its AdS minimum) is smaller than the naive parametric expectation, in agreement with the alternative derivation in appendix B. While this tends to exacerbate the ‘ D -term-suppression problem’ discussed earlier, the effect is already included in equation (2.22) and does not change the moderately optimistic conclusion drawn above.
- It should be clear from the above that in our scenario SUSY is broken at a high scale, $m_{3/2} \sim 10^{-3}$, avoiding the Kallosh-Linde problem [70] in a ‘trivial’ way. While it is interesting to investigate the possibility that, after reheating, a different moduli stabilisation mechanism takes over and low-scale SUSY is recovered [71, 72], we do not pursue this idea in the present paper.

3 Moduli Stabilisation - Hierarchical Setup

While we saw in the previous section that, within the Large Volume Scenario, it is possible to stabilise the Kähler moduli in an AdS minimum at exponentially large overall volume, we ran into trouble trying to uplift the minimum to Minkowski via a D -term potential: For $\mathcal{V} \simeq 1.7 \times 10^6$ the required size of W_0 is in tension with the D3-tadpole constraint. On the other hand, this clash is not expected to be present generically because in situations with more than two Kähler moduli there are further potentially large numbers to be considered. These are, in particular, the relative sizes of four-cycles and they may well improve the situation, depending on the precise intersection structure.

In fact, considering these more involved models has turned out to be essential for a completely unrelated reason: A more detailed analysis of the phenomenological requirements of fluxbrane inflation reveals that considering isotropic compactification manifolds is actually not enough. In fact, one of the promising outcomes of [19] was that in fluxbrane inflation the energy density of cosmic strings, which are formed upon brane recombination, can be controlled by the relative size of two four-cycles.

We start this section by quickly recalling the most important phenomenological impacts of cosmic strings in brane inflation and discuss the need for a hierarchy. This discussion

is followed by a brief review of how to stabilise the directions transverse to the overall volume in models with more than two Kähler moduli via string loop corrections. Finally, we include the D -term uplift and demonstrate that there is a region in parameter space in which all Kähler moduli are stabilised in a de Sitter minimum.

3.1 Cosmic Strings and the Need for a Hierarchy

At the end of brane inflation cosmic strings are formed generically [73–75]. As these objects carry some energy density they will leave an imprint on the CMB which one should be able to measure in principle. The fact that measurements have not revealed the presence of cosmic strings yet constrains the energy density of these objects. Due to the complicated nature of the bound state formed at the end of inflation, it is actually not immediately clear whether the produced cosmic strings are topologically stable (local) in our fluxbrane scenario. Since a detailed investigation of this interesting question is beyond the scope of this paper, we assume a worst case scenario of local cosmic strings.⁸ The resulting constraint can then be phrased as an upper bound on the value of the D -term ξ_- during inflation ($\xi_- \leq \xi_{\text{crit.}}$) [77], which reads

$$\frac{(\int_{\text{D7}} J \wedge \mathcal{F}_-)^2}{\frac{1}{2} \int_{\text{D7}} J \wedge J} \lesssim 8\pi^2 \frac{\alpha}{V_0} \xi_{\text{crit.}}^2. \quad (3.1)$$

We use the results from [78], which constrain the product $G\mu$ of the cosmic string tension μ and Newton’s constant G as $(G\mu)_{\text{crit.}} = \frac{1}{4}\xi_{\text{crit.}} \simeq 0.42 \times 10^{-6}$, i.e.

$$\frac{(\int_{\text{D7}} J \wedge \mathcal{F}_-)^2}{\frac{1}{2} \int_{\text{D7}} J \wedge J} \lesssim 9.4 \times 10^{-2}. \quad (3.2)$$

It is thus clear that our compactification manifold needs to have at least two ‘large’ four-cycles with hierarchically different volumes in order for the cosmic string bound to be satisfied. This leads us to consider hierarchical compactification proposals similar to the ones discussed for example in [41, 79]. The minimal modification of our previous setup is to investigate a scenario with three four-cycles, one of which is diagonal and small, supporting the non-perturbative effects.⁹ In the most generic situation the volume then takes the form (for an overview on our conventions see appendix A)

$$\mathcal{V} = \frac{1}{6} \sum_{i,j,k \in \{1,2\}} \kappa_{ijk} t^i t^j t^k + \frac{1}{6} \kappa_{333} (t^3)^3. \quad (3.3)$$

As the $(1,1)$ -form ω^3 is dual to a four-cycle which is contractible to a point, t^3 is negative.

We choose the following brane and flux setup: The pair of D7-branes is wrapped around the four-cycle dual to the $(1,1)$ -form ω^2 , while the brane flux is given by $\mathcal{F}_\pm = n_\pm \omega^1$. Thus, in order for the induced D3-brane charge ($\sim \int_{\text{D7}} \mathcal{F}_\pm \wedge \mathcal{F}_\pm$) to vanish we require $\kappa_{112} = 0$.

⁸ In the semilocal case [35–39], the constraints are weakened [76].

⁹ It would be important to search for concrete geometries, e.g. along the lines of [80]. However, for now we simply assume that this and other topological constraints can be met.

We now consider the limit $t^1 \gg |t^2| \gg |t^3|$.¹⁰ It turns out that for the Kähler metric $K_{T_i \overline{T_j}}$ to be positive definite in this limit we need $\kappa_{122} < 0$. Furthermore, requiring the D7-brane cycle to have positive volume ($\int_{D7} J \wedge J > 0$) we find $t_2 < 0$. The moduli of the dual 4-cycles $\tau_i = \partial \mathcal{V} / \partial t^i$ are given by

$$\tau_1 \simeq \frac{1}{2} \kappa_{111} (t^1)^2, \quad \tau_2 \simeq \kappa_{122} t^1 t^2, \quad \tau_3 = \frac{1}{2} \kappa_{333} (t^3)^2. \quad (3.4)$$

In these variables the overall volume can be written as

$$\mathcal{V} \simeq \frac{1}{3} \sqrt{\frac{2}{\kappa_{111}}} \tau_1^{3/2}. \quad (3.5)$$

It will be convenient to express all quantities in terms of τ_3 , \mathcal{V} and the quantity

$$x \equiv \frac{\tau_1}{\tau_2} = \frac{\kappa_{111}}{2\kappa_{122}} \frac{t^1}{t^2}, \quad (3.6)$$

which measures the hierarchy of the two ‘large’ four-cycles. For example, the constraints (2.5) and (3.2) can now be rewritten as

$$\mathcal{V}^{4/3} x = \frac{6^{2/3} \kappa_{111}^{1/3}}{4} \times 4.2 \times 10^8, \quad (3.7)$$

$$x \gtrsim 5.3 \, n_-^2 \kappa_{111}. \quad (3.8)$$

It is the purpose of section 3.3 to demonstrate that the Kähler moduli can actually be stabilised in a regime such that the above constraints are fulfilled. The way how x appears in (3.7) indicates that at large x the volume can be considerably smaller than in the setup without such hierarchical cycle volumes. This is one way in which the hierarchy between the four-cycle volumes alleviates the problem of a too large W_0 .

3.2 String Loop Corrections

As we saw in section 2.2 the interplay between α' -corrections to the Kähler potential and non-perturbative corrections to the superpotential allows for a minimum of the scalar potential with the overall volume \mathcal{V} stabilised at an exponentially large value and the small instanton four-cycle stabilised at $a_s \tau_s \sim \log(\mathcal{V}/|W_0|)$. However, for a model with more than two Kähler moduli there will be directions transverse to \mathcal{V} which remain flat. As was shown in [40, 41] these transverse directions may be stabilised by string loop corrections to the Kähler potential. In toroidal compactifications those corrections are well known [42–44]. Based on this work the authors of [45] conjectured that on a general Calabi-Yau manifold string loop corrections to the Kähler potential take the form

$$\begin{aligned} \delta K_{(g_s)} &= \delta K_{(g_s)}^{\text{KK}} + \delta K_{(g_s)}^{\text{W}} \\ &= \sum_{i=1}^{h_{1,1}} \frac{C_i^{\text{KK}}(U, \overline{U}) (a_{ij} t^j)}{\Re(S) \mathcal{V}} + \sum_{i=1}^{h_{1,1}} \frac{C_i^{\text{W}}(U, \overline{U})}{(b_{ij} t^j) \mathcal{V}}. \end{aligned} \quad (3.9)$$

¹⁰In section 3.3 we will show in detail that it is actually possible to stabilise the Kähler moduli in this regime.

These corrections originate from the exchange of Kaluza-Klein (KK) modes (with respect to a two-cycle $a_{ij}t^j$) between D7-branes and O7-planes, and of winding (W) modes of strings (along a two-cycle $b_{ij}t^j$ on which the D7-branes intersect). In the example of a toroidal compactification with $\mathcal{O}(1)$ values of the complex structure the functions $\mathcal{C}^{\text{KK},\text{W}}$ were calculated to be of the order 10^{-2} (see e.g. [44]).

Although the g_s -corrections coming from KK-modes are the leading corrections in the Kähler potential in terms of the scaling with the Kähler moduli, it was found [40] that in the F -term potential actually the α' -corrections are dominant. This feature is called *extended no-scale structure* and is crucial to ensure the overall consistency of the approach. Furthermore, as the g_s -corrections depend not only on the overall volume \mathcal{V} but also on the two-cycle moduli t^i , it is intuitively clear that these corrections potentially stabilise the flat directions.

Following [45] we will assume the g_s -corrections in our scenario to take the form

$$\delta K_{(g_s)} = \frac{g_s}{\mathcal{V}} (C_1^{\text{KK}} t^1 + C_2^{\text{KK}} t^2) + \frac{1}{\mathcal{V}} \left(\frac{C_1^{\text{W}}}{t^1} + \frac{C_2^{\text{W}}}{t^2} \right). \quad (3.10)$$

From these terms one can compute the corresponding corrections to the scalar F -term potential as (cf. [40])

$$\delta V_{(g_s)} = V_{0,F} \frac{W_0^2}{\mathcal{V}^2} \left\{ \frac{g_s^2}{\tau_1^2} \left(3 (C_1^{\text{KK}})^2 + \frac{3}{2} \frac{\kappa_{111}}{|\kappa_{122}|} (C_2^{\text{KK}})^2 \right) - \frac{2}{\mathcal{V}} \left(\frac{C_1^{\text{W}}}{t^1} + \frac{C_2^{\text{W}}}{t^2} \right) \right\} \quad (3.11)$$

$$= V_{0,F} \frac{W_0^2}{\mathcal{V}^3} \frac{1}{\mathcal{V}^{1/3}} \{ \mathcal{A} g_s^2 + \mathcal{B} + \mathcal{C} x \} \quad (3.12)$$

with

$$\begin{aligned} \mathcal{A} &= \frac{2^{2/3}}{3^{1/3} \kappa_{111}^{2/3}} \left((C_1^{\text{KK}})^2 + \frac{\kappa_{111}}{2 |\kappa_{122}|} (C_2^{\text{KK}})^2 \right) > 0, \\ \mathcal{B} &= -2 C_1^{\text{W}} \left(\frac{\kappa_{111}}{6} \right)^{1/3}, \quad \mathcal{C} = \frac{4 C_2^{\text{W}} |\kappa_{122}|}{(6 \kappa_{111}^2)^{1/3}} \end{aligned} \quad (3.13)$$

and $x \equiv \tau_1/\tau_2$ as before. These string loop corrections stabilise x . The precise way in which this happens is presented in the next section.

3.3 Moduli Stabilisation in a Modified Large Volume Scenario

In the hierarchical setup, turning on fluxes $\mathcal{F}_\pm = n_\pm \omega^1$ on the D7-brane pair wrapping the four-cycle with modulus τ_2 of the manifold will induce a D -term potential [19, 48, 49]

$$V_D^\pm = \frac{1}{16\pi\mathcal{V}^2} \frac{(\int_{\text{D7}} J \wedge \mathcal{F}_\pm)^2}{\frac{1}{2} \int_{\text{D7}} J \wedge J} \simeq \frac{1}{16\pi\mathcal{V}^2} \frac{n_\pm^2 \kappa_{111}}{2x} \quad (3.14)$$

in the effective theory. In the last step we used $t^1 \gg |t^2|$. This term in the scalar potential is enhanced by powers of the overall volume \mathcal{V} as compared to (3.12). For some fixed \mathcal{V} the D -term will drive x to large values at which the term $\sim x$ in $\delta V_{(g_s)}$ will become important. It is this feature (enhancement of $\delta V_{(g_s)}$ and suppression of V_D by a large x) which, together

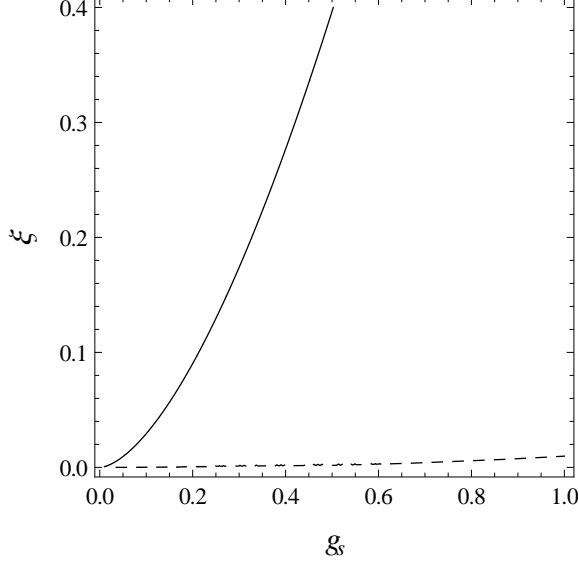


Figure 2. The cosmic string bound is satisfied in the region above the dashed line while the CMB amplitude has the right size on the solid line.

with the mechanism discussed below equation (3.7), improves the situation as compared to the model of section 2.3 in which W_0 had to be chosen very large.

We now turn to the minimisation of the potential in x -direction. In view of our ignorance concerning the prefactors \mathcal{A} , \mathcal{B} , and \mathcal{C} , we will *assume* in the sequel that \mathcal{C} is positive such that x will be stabilised. The relevant term in (3.12) is given by

$$\delta V_{(g_s)} \simeq \frac{1}{16\pi\mathcal{V}^2} \frac{g_s W_0^2}{\mathcal{V}^{4/3}} \mathcal{C} x. \quad (3.15)$$

This term together with (3.14) will stabilise x at the value

$$x_{\min.} = \frac{n\sqrt{\kappa_{111}}\mathcal{V}^{2/3}}{\sqrt{2g_s\mathcal{C}W_0}} \quad (3.16)$$

where again $n = \sqrt{n_+^2 + n_-^2}$. Plugging this result back into (3.14) and (3.16), the full scalar potential reads

$$V(\mathcal{V}) \simeq \frac{g_s W_0^2}{16\pi\mathcal{V}^3} \left(\frac{\xi\gamma}{g_s^{3/2}} - \frac{c\beta^2}{4\alpha d_s^{3/2}} \log^{3/2} \left(\frac{2\alpha\mathcal{V}}{c\beta W_0} \right) + \frac{n\sqrt{2\kappa_{111}\mathcal{C}}}{\sqrt{g_s W_0}} \mathcal{V}^{1/3} \right). \quad (3.17)$$

From now on we can proceed in complete analogy to the analysis of section 2.2. Vanishing of the potential at its minimum gives (to leading order in $\log \left(\frac{4a_s A_s}{3c} \frac{\mathcal{V}}{|W_0|} \right)$)

$$\frac{a_s}{g_s} \left(\frac{\xi}{2c} \right)^{2/3} = \log \left(\frac{4a_s A_s}{3c} \frac{\mathcal{V}}{|W_0|} \right) \quad (3.18)$$

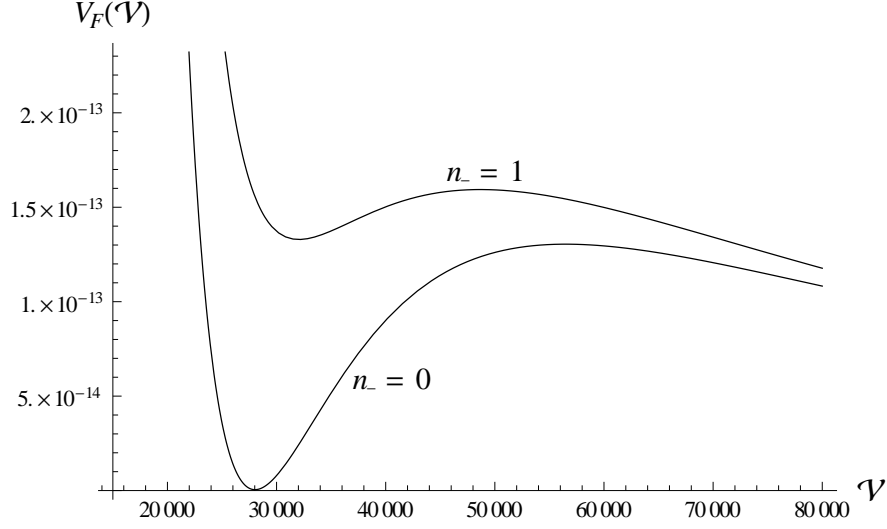


Figure 3. Plot of (3.17) for $n_+ = 12$, $n_- \in \{0, 1\}$. The numerical value for ξ corresponding to a Minkowski minimum (for $n_- = 0$) relates to the analytic first-order result in (3.17) as $\xi_{\text{numeric}} = 0.3622 \cdot \xi_{\text{analytic}}$.

which, together with the vanishing of the first derivative, implies

$$W_0 = \frac{2^{2+3/4}}{3^4} \frac{a_s}{c\sqrt{A_s}} \left(\frac{n_+^2 \kappa_{111} \mathcal{C}}{g_s} \right)^{3/4} \frac{e^{\frac{1}{2}a_s \tau_s}}{\tau_s^{3/4}}, \quad (3.19)$$

where τ_s is again given by (2.20). Plugging this back into (3.18) gives an expression for \mathcal{V} in terms of g_s , ξ , etc.

Thus, by minimising the potential (3.17) in a Minkowski minimum we have expressed all quantities in terms of g_s , ξ , A_s , \mathcal{C} , n_+ . The next step is to implement the constraints (3.7) and (3.8). We choose¹¹ $n_+ = 12$, $A_s = 1$, $\mathcal{C} = 0.02$, $\kappa_{111} = 5$, $c = \sqrt{2}/3$, and plot the constraints in the g_s - ξ -plane (see figure 2). Choosing $g_s = 0.5$, equation (3.7) implies $\xi = 0.4$ (which corresponds to a threefold X_3 with Euler characteristic $\chi(X_3) \simeq 164$) and

$$W_0 = 4.3 \times 10^2, \quad (3.20)$$

$$\mathcal{V} = 2.8 \times 10^4, \quad (3.21)$$

$$x = 4.1 \times 10^2. \quad (3.22)$$

This also implies $\sqrt{g_s/2} W_0 = 2.1 \times 10^2$ which requires $\chi(X_4) \simeq 1.1 \times 10^6$ (cf. the discussion at the end of section 2). Fourfolds with such Euler characteristics are known in the literature (see e.g. [47]). Furthermore, in view of (2.21) we find $m_{3/2}/m_{\text{KK}} \simeq 0.8$. While we are aware that with such a large $m_{3/2}$ our approach is only marginally consistent we take

¹¹As motivated in footnote 5 there is a lower bound on n_+ which arises from the requirement of preserving the local minimum for \mathcal{V} after the uplift to de Sitter. An analytical estimate for this bound, performed in appendix C, gives $n_+ \geq 8$. Figure 3 shows that for a more conservative choice of $n_+ = 12$ the minimum is indeed still present after including V_D^- .

the improvement of the situation compared to the ‘warm-up’-model of section 2 as an encouraging sign and a reason to further investigate hierarchical large volume models with D -term uplifts [24].

4 Flat Directions for the Inflaton

In this section we give an overview over possible inflaton mass corrections from the F -term scalar potential. While, for most cases, we indicate ways in which these mass terms can be absent or suppressed, we stress that large parts of the subject are still work in progress and will be discussed more thoroughly in a further publication [24].

The most direct way in which the D7-brane modulus ζ can enter the F -term potential is through a direct appearance in the tree-level superpotential [49, 81–85]

$$W_{\text{brane}} = \int_{\mathcal{C}_5} \Omega \wedge \tilde{\mathcal{F}}. \quad (4.1)$$

Here, \mathcal{C}_5 is a five-chain ending on the brane divisor Σ , Ω is the holomorphic $(3,0)$ -form pulled back to \mathcal{C}_5 , and $\tilde{\mathcal{F}}$ is the brane flux \mathcal{F} continued to the five-chain. For $\tilde{\mathcal{F}}|_{\Sigma} \in H^{(1,1)}(\Sigma)$ the wedge product in (4.1) hence vanishes. Thus it is sufficient to choose $\tilde{\mathcal{F}}$ such that $\tilde{\mathcal{F}}|_{\Sigma}$ is in the image of $H^2(X_3)$ under pullback to Σ : Since for a Calabi-Yau $H^2(X_3) = H^{1,1}(X_3)$, such fluxes will always give identically vanishing W_{brane} . The flux we consider in this paper is of that kind because it is such flux that generates a D -term potential.

For completeness we furthermore recall from the discussion after equation (2.7) that non-perturbative effects from fluxed D3-brane instantons can introduce an explicit inflation dependence of the superpotential, which can be avoided by suitable constraints on the geometry of the instanton divisors.

Nevertheless, even if we can avoid a direct appearance of the brane modulus ζ in W , the fact that the tree-level superpotential for the complex structure moduli is stabilised at some $W_0 \neq 0$ can lead to a large inflaton mass: Assuming for simplicity a minimal Kähler potential $k(\zeta, \bar{\zeta}) = \zeta \bar{\zeta}$ we find that roughly

$$m_{\zeta} \simeq m_{3/2} \quad (4.2)$$

(cf. the discussion at the end of section 2), which is much larger than the Hubble scale and thus spoils slow-roll inflation generically.

The situation is different if the ζ -moduli space possesses a shift-symmetry such that, for example, $k(\zeta, \bar{\zeta}) \equiv k(\zeta + \bar{\zeta})$, i.e. the Kähler potential is independent of the imaginary part of ζ , which thus remains a flat direction. The potential role of shift symmetries protecting the inflaton mass from dangerous F -term contributions was also discussed previously in the context of D3/ $\overline{\text{D3}}$ and D3/D7 inflation (i.e. for mobile D3-branes) in [18, 23, 50, 51]. However, in these cases one faces some concerns: Usually, in these models inflation proceeds as the D3-brane moves in the radial direction of a warped deformed conifold. The Kähler potential of the conifold, however, possesses no radial shift symmetry. More generally, as noted in [23], isometries of the moduli space of D3-positions are not generically present in

Type IIB compactifications. In fact, the moduli space of D3-positions, which is nothing but the compactification manifold itself, cannot exhibit a shift symmetry if it has the full $SU(3)$ holonomy. Therefore, only in special examples with manifolds of reduced holonomy, such as compactifications on $K3 \times T^2/\mathbb{Z}_2$, one can hope to find a Kähler potential with the desired feature. On the other hand, fluxbrane inflation is in a different position: Even in flat space the leading order potential is flat enough to easily give rise to an inflationary epoch which lasts 60 e-foldings. Furthermore, the moduli space of D7-positions is not the compactification manifold but rather some other Kähler manifold, the Kähler potential of which may well exhibit shift symmetries also in more general cases. Therefore, while fluxbrane inflation is a scenario in which one can actually make use of the shift symmetries in the above examples of toroidal and $K3 \times T^2$ orientifolds, one can even hope to find suitable compactifications beyond these simple models.

In fact, even if the moduli space of D7-brane positions possesses no shift-symmetry generically, there can still be regions in parameter space where an approximate shift-symmetry exists. Here we summarise our present understanding within the ongoing investigation [24]: Some of the D7-brane moduli correspond, on the mirror-dual type-IIA orientifold with D6-branes, to Wilson-line moduli. Up to instanton and loop-corrections, the latter enjoy a shift-symmetry originating in the gauge-symmetry of the D6-world-volume gauge theory. We can hence expect this shift symmetry to be present in our setting if, in addition to being at large volume, we also insist in being near the large-complex-structure point. We finally note the possible interest in shift-symmetries of this type in the context of Higgs-physics with a high-scale SUSY breaking [86–88].

It is interesting to consider the global properties of the ζ -moduli space. As the moduli space of the axio-dilaton S is non-trivially fibered over the former, the Kähler potential has the structure

$$K \supset -\log(S + \bar{S} - k(\zeta, \bar{\zeta})). \quad (4.3)$$

Typically, the moduli space of the D7-brane modulus ζ is covered by a set of coordinate patches with appropriate transition functions. Generically, the Kähler potential $k(\zeta, \bar{\zeta})$ on the D7 moduli space is not globally defined but rather undergoes a transformation $k(\zeta, \bar{\zeta}) = k'(\zeta', \bar{\zeta}') + f(\zeta') + \bar{f}(\bar{\zeta}')$ for a transition function $\zeta = \zeta(\zeta')$. For the full Kähler potential K in (4.3) to remain invariant, this transformation has to be absorbed by a simultaneous redefinition of the axio-dilaton: $S = S' + f(\zeta')$. Invariance of K implies invariance of the superpotential: $W(S) = W'(S', \zeta')$. In other words, W can not be independent of the brane position moduli in all patches. As W is holomorphic there is then no chance of having a manifest shift-symmetry in all patches. However, all we need is to find a flux choice for which the D7-brane coordinate does not appear in W in one particular coordinate patch. After a change of coordinates it will be some combination of the axio-dilaton S and the brane modulus ζ which is a flat direction in the scalar potential. In [24] we will demonstrate this feature in the example of a $K3 \times T^2/\mathbb{Z}_2$ compactification in detail. The ambiguity of having $W(S)$ or $W(S, \zeta)$, depending on the coordinate patch, is related to the ambiguity in the definition of brane or bulk fluxes. This, in turn, has to do with the $SL(2, \mathbb{Z})$ monodromy affecting S and (F_3, H_3) at 7-brane positions [46]. Pertinent

investigations of the 7-brane superpotential include the recent [89–95].

The analog of this issue in the case of inflation with mobile D3-branes has been discussed in detail in the literature [23, 96]. The superpotential depends non-perturbatively on the volume modulus (which, in this setup, plays a role analogous to S in the case of fluxbrane inflation). The gauge kinetic function, which enters this non-perturbative term, receives one-loop corrections which depend on the D3-brane position. For a toroidal compactification these corrections were analysed in [96] and shown to respect a discrete shift-symmetry, reflecting the compactness of the torus. No continuous shift-symmetry is preserved by these corrections. Crucially, there is no mechanism by which the appearance of the D3-brane coordinate in the superpotential can be avoided.

Finally, we note that the string loop corrections, which were used in section 3.2 to stabilise the relative size of the two large four-cycles, generically depend on open string moduli. The precise form of this dependence in toroidal models can in principle be extracted from [43, 44]. The relevance of these corrections for the flatness of the inflaton potential is presently under investigation [24].

5 Consistency of the Effective Theory

While the question of a potential inconsistency of (constant) FI terms in supergravity is not a novel issue (see, e.g. [35, 52]), it has attracted an increased amount of interest more recently [53–59]. Given that D -term inflation in its original form [5, 6] relies on the presence of a (constant) FI term and that the existence of consistent gravity models with this feature is doubtful, we find it necessary to devote a section of our paper to this issue. For example, the arguments above have led the authors of [97] to conclude that D3/D7 inflation as well as fluxbrane inflation are subject to rather stringent constraints. As we will explain, we believe that our construction can not come into conflict even with the most stringent no-go theorems concerning FI terms that are being debated.

The viability of D -term inflation in view of supergravity constraints on FI terms has also been discussed in [35]. However, since our perspective and (part of) our conclusions are different, we believe that it is worthwhile to revisit this issue.

5.1 Issues in String D -Terms

Recall that the D -term in supergravity is given in general by [98]

$$\xi = iK_i X^i(z), \quad (5.1)$$

where K is the Kähler potential and $X(z)$ is the holomorphic Killing vector generating the (gauged) isometry of the moduli space. We denote the coordinates z^i on that moduli space collectively by z . The D -term potential then reads

$$V_D = \frac{g_{\text{YM}}^2}{2} \xi^2. \quad (5.2)$$

The consistency question alluded to above is, roughly speaking, under which circumstances one may write

$$\xi = iK_i X^i(z) + \xi_0 \quad (5.3)$$

for some constant $\xi_0 \neq 0$. For our purposes, the precise answer to this question is, in fact, irrelevant. We are only interested in string-derived models and hence for us it is sufficient to know that no such constant arises in the low-energy limit of string compactifications [99–102] (at least there are no such examples). Moreover, as we will work out in more detail momentarily, our D -term potential is described by the (undebated) part $iK_i X^i(z)$. Since this has given us a viable model of inflation, one might think that the ‘FI term-issue’ in fluxbrane inflation is thus closed.

Things are not quite as simple, though. Given that the D -term potential drives inflation, the moduli in $iK_i X^i(z)$ must be stabilised. In fact, this was the main theme of the present investigation. Hence one might expect to encounter, somewhere between the moduli-stabilisation scale and the SUSY-breaking scale, an effective theory with constant FI term. This would not only be potentially inconsistent, it turns out to be technically impossible in models where no FI term is originally present [35, 53, 55]. How can any stringy version of D -term inflation then exist? The answer suggested in [35] was to have a *small* F -term potential giving a *large* mass to the relevant moduli, which might be possible with a special choice of Kähler potential. Jumping ahead, our answer is different: In our scenario the SUSY breaking scale is enhanced as compared to the scale at which the Kähler moduli are stabilised. This fact is easy to understand: As we will confirm momentarily, the Kähler moduli naturally have masses $m_\tau^2 \sim V_D \sim \mathcal{V}^{-2}$. On the other hand the gravitino mass is given by $m_{3/2}^2 \sim W_0^2/\mathcal{V}^2$. Recall that, as a result of the approximate no-scale structure in the F -term potential and the requirement $V_D \sim V_F$, we work at parametrically large W_0 . Therefore, the gravitino mass is parametrically larger than m_τ . Due to this particular hierarchy of scales our model avoids the above constraints ‘trivially’. We will come back to this fact at the end of this section.

5.2 Moduli Masses in Fluxbrane Moduli Stabilisation

We first put our D -term potential in the standard $N = 1$ supergravity form following [30]: Let D_j be a divisor with dual 2-form $[D_j]$ and let the fluxed D7-brane be wrapped on a divisor $D_{\mathcal{F}}$. The four-cycle modulus τ_j parametrising the size of D_j gets charged under the $U(1)$ on the D7-brane if the flux living on the intersection $D_j \cap D_{\mathcal{F}}$ is non-vanishing. Since the symmetry which is gauged is an axionic shift symmetry, the corresponding Killing vector is just $X_j = iq_j$ with q_j the charge of τ_j . Thus, in the low-energy effective action a D -term

$$\xi = -q_j K_{\tau_j} \tag{5.4}$$

appears, where K is the Kähler potential. The charge q_j depends on the flux [30]: $q_j \sim \int_{D_{\mathcal{F}}} [D_j] \wedge \mathcal{F}$. In particular, one can show that with this input that (5.4) is equivalent to [48, 49]

$$\xi = \frac{1}{4\pi} \frac{\int_{D_{\mathcal{F}}} J \wedge \mathcal{F}}{\mathcal{V}}, \tag{5.5}$$

which was used in section 2 and section 3.

In the simple two-Kähler moduli example at the end of section 2, we wrapped the D-brane on D_b and chose a flux $\mathcal{F} = n[D_b]$. As κ_{bbb} is non-zero, τ_b is charged under the

$U(1)$, generating a D -term of the form (5.5). The potential terms relevant for the mass of τ_b are (2.12) and (2.14). The corresponding Kähler potential is

$$K = -3 \log \tau_b + \dots \quad (5.6)$$

For simplicity we compute the mass in the Minkowski minimum (the result changes only by an $\mathcal{O}(1)$ factor when going to de Sitter). Working in addition to leading order in $\log\left(\frac{2\alpha\mathcal{V}}{c\beta W_0}\right)$, we find

$$m_{\tau_b}^2 = \frac{1}{K_{\tau_b\tau_b}} \partial_{\tau_b}^2 V \Big|_{\tau_b^{\min.}} = \frac{3}{4} V_D \Big|_{\tau_b^{\min.}}, \quad (5.7)$$

where V_D is given in (2.14).

This can be compared to the mass of the vector boson which gauges the axionic shift-symmetry. The relevant terms in the Lagrangian are

$$\mathcal{L} \supset -\frac{1}{4g_{\text{YM}}^2} F^2 + K_{i\bar{j}} D_\mu z^i \overline{D^\mu z^{\bar{j}}}, \quad (5.8)$$

with

$$D_\mu z^i = \partial_\mu z^i - A_\mu X^i(z). \quad (5.9)$$

Thus, using also (5.1) and (5.2) the gauge boson mass is

$$m_V^2 = 2g_{\text{YM}}^2 K_{T_b\bar{T}_b} |X^{T_b}|^2 = \frac{4}{3} V_D \Big|_{\tau_b^{\min.}}. \quad (5.10)$$

We conclude that the masses of vector boson and corresponding Kähler modulus are of the same order of magnitude. Moreover, since $V_D \sim H^2$, both masses are related to the Hubble scale. While the purist might object both to calling this D -term inflation (since $V_F \sim V_D$) and to calling it single-field inflation (since $m_{\tau_b} \sim H$), we are, for the time being, satisfied with this outcome.

Finally, an analysis similar to the one performed in appendix C but for the simple model of section 2 gives

$$\frac{\delta V_D}{V_D} \lesssim \frac{2}{3^3} \quad (5.11)$$

where δV_D represents the additional energy density due to $\mathcal{F}_- \neq 0$ during inflation. This implies that the Hubble scale during inflation is given by $H_{\text{infl.}} \lesssim 0.07 \times V_D$. Therefore, τ_b is actually somewhat heavier than the parametric analysis above suggests ($m_{\tau_b}/H_{\text{infl.}} \gg 1$) and its dynamics can be disregarded during inflation.

We note that our result can be understood more generally (see e.g. [35, 53]): In unbroken SUSY the mass of the vector and the mass of the volume modulus are the same because the $U(1)$ is higgsed by the axionic scalar from the volume superfield (in our case T_b). This equality can only be lifted by SUSY breaking. Thus, if the mass of the volume modulus is stabilised at a scale much above the vector mass, supersymmetry must be broken at this high scale. As result, there can be no energy domain where τ_b is consistently integrated out while the gauge boson is kept as a dynamical degree of freedom in a supersymmetric

theory. In other words, as mentioned earlier, even an *effectively* constant FI term can not arise.

In our specific setting (at least in the toy model version of section 2), we have $H^2 \sim m_V^2 \sim m_{\tau_b}^2 \sim V_D \sim V_F$, as demonstrated above. By contrast, $m_{3/2}^2$ is much larger. This is due to the (approximate) no-scale cancellation which makes V_F smaller than its naive parametrical size $|e^K W_0^2|$. Hence, we are indeed more than safe from any regime with unbroken SUSY and an effectively constant FI term. Of course, the analysis in the present section dealt just with the toy model of section 2. An analogous discussion of the hierarchical model of section 3 is qualitatively similar but much more involved. While the various ‘low-lying’ mass scales from H to m_V are now somewhat different, the much larger size of $m_{3/2}$ is a generic feature. It will continue to ensure that SUSY is broken before the moduli are frozen.

6 Conclusions

We have studied moduli stabilisation in fluxbrane inflation. In this scenario, the role of the inflaton is played by the relative position of two D7-branes, attracted towards each other by non-supersymmetric gauge flux. This can be viewed as a variant of D -term inflation where, as is well-known, a very small FI term is required to reproduce the observed magnitude of CMB fluctuations. In our context, this implies large brane volume and hence, in general, large compactification volume ($\sim 10^6$ in string units).

We therefore work in the Large Volume Scenario, where the interplay of α' - and instanton corrections stabilises an exponentially large overall volume. The resulting non-SUSY AdS-vacuum is then uplifted by a D -term to realise the inflationary almost-de-Sitter phase. Stability requires the F -term to be roughly of the same size as the D -term, which we ensure by using a parametrically large flux-potential W_0 . This entails two problems: a dangerously large (close to the KK-scale) gravitino mass and an enormous D3 tadpole (in excess of the largest known fourfold Euler numbers). Moreover, as is common in D -term inflation in general, the large cosmic string scale is problematic.

Fortunately, a rather natural generalisation of the simplest construction can resolve all of the above issues: We explicitly include a third Kähler modulus (the minimum in large volume models being two). In this case, two of the Kähler moduli are stabilised at exponentially large values. Their relative size is fixed by the interplay of loop corrections and the D -term. This introduces a new parameter – the ratio of two four-cycle volumes – which can take a small value ($\sim 1/400$) in a concrete model. In appropriate settings, this smallish number suppresses the D -term (thereby lowering the required value of W_0) as well as the cosmic string tension. Thus, while we do not provide an explicit Calabi-Yau orientifold and brane configuration, we are able to demonstrate the phenomenological viability of our scenario with reasonable assumptions concerning topological data and loop-correction coefficients.

To realise our flat inflaton potential, we had to assume a choice of 3-form flux by which the relevant D7-brane modulus is not stabilised. Nevertheless, a generic Kähler potential for the brane moduli leads, in presence of non-zero F -terms, to the familiar η -

problem. Referring to a forthcoming publication for details, we propose to overcome this problem as follows: By mirror symmetry, some of the D7 position moduli correspond to D6 Wilson lines. The latter enjoy a shift-symmetric Kähler potential at large Type IIA volume. Hence, we expect a shift-symmetric inflationary Kähler potential in Type IIB at large complex structure (in addition to large volume). It is far from obvious to which extent this idea will survive the loop corrections which we also use. This is presently under investigation.

Finally, we commented on certain consistency issues in the context of the recent ‘supergravity FI term debate’. We have no proper (‘constant’) FI term but rather a conventional D -term potential (or ‘field-dependent FI term’). The latter comes from the gauging of an isometry of the Kähler moduli space. We show explicitly that, in our construction, the vector boson mass, the Kähler modulus mass and the Hubble scale are parametrically of the same order of magnitude. Hence, the potential problem of an ‘effectively constant FI term’ at some intermediate energy scale (which would hint at some hidden inconsistency) does not arise. Thus, D -term inflation (at least in our definition, i.e. allowing for comparable stabilising F -terms) is well and alive independently of the existence of constant FI terms.

So far, we can only view our investigation as a small step in the ongoing struggle to eventually establish inflation in string theory (or rule it out). For the future of our proposal, much will depend on our ability to understand how D7-brane moduli enter the Kähler potential and superpotential at subleading order.

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Appendix

A Definitions and Conventions

In this appendix we collect definitions and conventions used in this article.

The string length is given by $\ell_s = 2\pi\sqrt{\alpha'}$. We take the transformation from string to 10d Einstein frame to be

$$g_{MN}^S = e^{\frac{\phi}{2}} g_{MN}^{\text{E10}} \quad (\text{A.1})$$

and we use $g_s = \langle e^\phi \rangle$, $\tau = e^{-\phi} + iC_0$.¹² The volume \mathcal{V} of the compactification space is

¹²Note that the relation between τ defined here and the supergravity variable S used e.g. in (1.2) is non-trivial [48, 49].

defined by

$$\mathcal{V} = \frac{1}{\ell_s^6} \int d^6x \sqrt{g_6^{\text{E10}}}. \quad (\text{A.2})$$

The transformation from 10d Einstein frame to 4d Einstein frame is given by

$$g_{\mu\nu}^{\text{E10}} = \frac{1}{\mathcal{V}} g_{\mu\nu}^{\text{E4}}. \quad (\text{A.3})$$

The four-dimensional Planck mass is then given by

$$M_p^2 = \frac{4\pi}{\ell_s^2}. \quad (\text{A.4})$$

It is set to one in all 4d-field-theory formulae. The Kähler form J in the Einstein frame is expanded in a basis of $(1,1)$ -forms $J = t^i \omega^i$, such that the volume of the manifold can be written as

$$\mathcal{V} = \frac{1}{6} \int J \wedge J \wedge J = \frac{1}{6} \kappa_{ijk} t^i t^j t^k. \quad (\text{A.5})$$

The volumes of the four-cycles dual to the $(1,1)$ -forms ω^i are defined via $\tau_i = \partial \mathcal{V} / \partial t^i$.

B F -Term Scalar Potential

The main purpose of this appendix is to analyse the F -term potential discussed in section 2.2. Such a potential arises in the original Large Volume Scenario as proposed in [25] as well as in more elaborate versions thereof [41], which is the case of interest for us.

Starting point is the expression¹³

$$\begin{aligned} V_F = & \frac{e^K}{8\pi} \left[K^{s\bar{s}} a_s^2 |A_s|^2 e^{-2a_s \tau_s} \right. \\ & - a_s K^{s\bar{p}} \partial_{\bar{p}} K e^{-a_s \tau_s} \left\{ W \bar{A}_s e^{ia_s b_s} + \bar{W} A_s e^{-ia_s b_s} \right\} \\ & \left. + \frac{3\xi |W_0|^2}{4g_s^{3/2} \mathcal{V}} \right] \end{aligned} \quad (\text{B.1})$$

which is obtained after plugging (2.7) and (2.8) into the standard supergravity formula for the F -term potential

$$V = \frac{e^K}{8\pi} \left(K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2 \right), \quad (\text{B.2})$$

expanding in leading order in $1/\mathcal{V}$, and neglecting all terms $\propto e^{-a_p \tau_p}$, $p \neq s$ (cf. [103]).

Consider the second line of equation (B.1). We can rewrite the term in the brackets as

$$2|W_0| |A_s| \cos(\arg(W_0) - \arg(A_s) + a_s b_s). \quad (\text{B.3})$$

Furthermore, using the identity

$$K^{s\bar{p}} \partial_{\bar{p}} K = -2\tau_s + \text{higher orders in } 1/\mathcal{V} \quad (\text{B.4})$$

¹³There seems to be a disagreement in the literature concerning the overall prefactor of the supergravity potential (see [62] and [26]). However, this factor is irrelevant for our purposes as we can simply choose to work with a differently normalised W_0 .

(cf. e.g. [40]) it is clear that minimising V_F with respect to the axion b_s will give $\cos(\dots) \rightarrow -1$ in (B.3) and thus the second term in (B.1) becomes $-4a_s\tau_s e^{-a_s\tau_s}|W_0||A_s|$.

Now we turn to the first line in (B.1): Using $\mathcal{V}(\tau_p) = \tilde{\mathcal{V}}(\tau_{p \neq s}) - c\tau_s^{3/2}$ we find

$$K_{s\bar{s}} \simeq \frac{3}{8} \frac{c}{\mathcal{V}\tau_s^{1/2}}, \quad K_{p\bar{s}} \simeq -\frac{3}{4} \frac{c(\partial_p \mathcal{V})\tau_s^{1/2}}{\mathcal{V}^2}, \quad (\text{B.5})$$

i.e. $K_{p\bar{q}}$ is block-diagonal in leading order in $1/\mathcal{V}$. Therefore, $K^{s\bar{s}} \simeq \frac{8}{3} \frac{\mathcal{V}\tau_s^{1/2}}{c}$ in leading order. Combining all the results we find

$$V_F = V_{0,F} \left(\frac{\alpha\sqrt{\tau_s}e^{-2a_s\tau_s}}{c\mathcal{V}} - \frac{\beta|W_0|\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\gamma\xi|W_0|^2}{g_s^{3/2}\mathcal{V}^3} \right) \quad (\text{B.6})$$

with

$$V_{0,F} = \frac{g_s e^{K_{cs}}}{16\pi}, \quad \alpha = \frac{8a_s^2|A_s|^2}{3}, \quad \beta = 4a_s|A_s|, \quad \gamma = \frac{3}{4}. \quad (\text{B.7})$$

We now compute the large volume minimum of (B.6). To this end, we evaluate

$$\frac{\partial V}{\partial \mathcal{V}} = 0, \quad \frac{\partial V}{\partial \tau_s} = 0. \quad (\text{B.8})$$

The first condition gives

$$\mathcal{V} = \frac{\beta|W_0|c\sqrt{\tau_s}e^{a_s\tau_s}}{\alpha} \left(1 - \sqrt{1 - \frac{3\alpha\gamma\xi}{g_s^{3/2}c\beta^2\tau_s^{3/2}}} \right) \quad (\text{B.9})$$

while the second equation reads

$$\frac{\mathcal{V}\alpha e^{-a_s\tau_s}}{\beta c|W_0|\sqrt{\tau_s}} = \frac{1 - a_s\tau_s}{\frac{1}{2} - 2a_s\tau_s}. \quad (\text{B.10})$$

This can be rearranged using (B.9):

$$1 - \sqrt{1 - \frac{3\alpha\gamma\xi}{g_s^{3/2}c\beta^2\tau_s^{3/2}}} = \frac{1 - a_s\tau_s}{\frac{1}{2} - 2a_s\tau_s}. \quad (\text{B.11})$$

For $a_s\tau_s \gg 1$, this equation simplifies and we obtain to leading order

$$\tau_s = \frac{1}{g_s} \left(\frac{4\gamma\alpha\xi}{c\beta^2} \right)^{2/3} = \frac{1}{g_s} \left(\frac{\xi}{2c} \right)^{2/3}, \quad (\text{B.12})$$

$$\mathcal{V} = \frac{\beta|W_0|c\sqrt{\tau_s}e^{a_s\tau_s}}{2\alpha} = \frac{\beta|W_0|c}{2\alpha g_s^{1/2}} \left(\frac{\xi}{2c} \right)^{1/3} e^{\frac{a_s}{g_s} \left(\frac{\xi}{2c} \right)^{2/3}}. \quad (\text{B.13})$$

From the above analysis and the definition of ξ below equation (2.8) it is clear that the requirement $a_s\tau_s \gg 1$ is true as long as $-\chi(X_3) \gg \frac{4c}{\zeta(3)}(2\pi g_s)^{3/2}$. Since $\chi(X_3) = 2(h^{(1,1)} - h^{(2,1)})$ and $h^{(1,1)} = 2$, this sets a lower bound on the number of complex structure moduli which is, however, rather easy to satisfy for all perturbative values of g_s . For

instance, the manifold $\mathbf{P}_{[1,1,1,6,9]}^4$ of [25], on which the large volume scenario was first constructed explicitly, has 272 complex structure moduli.

To compute the value of the potential V_F at the minimum, we solve (B.10) for \mathcal{V} and plug it into (B.6) to obtain

$$V_F = V_{0,F} \frac{\alpha^2}{\beta c^2 |W_0|} e^{-3a_s \tau_s} \left(X - X^2 + \frac{\alpha \gamma \xi}{\beta^2 c g_s^{3/2}} \tau_s^{-3/2} X^3 \right), \quad (\text{B.14})$$

where

$$X = \left(\frac{\frac{1}{2} - 2a_s \tau_s}{1 - a_s \tau_s} \right). \quad (\text{B.15})$$

Rewriting (B.11) yields

$$\frac{\alpha \gamma \xi}{\beta^2 c g_s^{3/2}} \tau_s^{-3/2} = \frac{1}{3} (2X^{-1} - X^{-2}) \quad (\text{B.16})$$

and therefore

$$V_F = V_{0,F} \frac{\alpha^2}{\beta c^2 |W_0|} e^{-3a_s \tau_s} \left(\frac{2}{3} X - \frac{1}{3} X^2 \right) \approx V_{0,F} \frac{\alpha^2}{\beta c^2 |W_0|} e^{-3a_s \tau_s} \left(-\frac{1}{a_s \tau_s} \right) \quad (\text{B.17})$$

for $a_s \tau_s \gg 1$. Remarkably, the minimum value of the F -term potential is suppressed by a factor of $a_s \tau_s$ relative to its natural value. Using (B.12) and (B.13) we finally obtain

$$V_F = -\frac{3M_p^4 \sqrt{g_s} e^{\mathcal{K}_{cs}} c |W_0|^2}{128\pi^2} \frac{c |W_0|^2}{\mathcal{V}^3} \left(\frac{\xi}{2c} \right)^{1/3} \quad (\text{B.18})$$

where we chose to explicitly write \mathcal{V} and $|W_0|$ in order to get a feeling for the size of the F -term potential in its minimum. It is clear, that via (B.12) and (B.13) one can express the minimum value solely in terms of ξ , g_s , c etc.

C Lower Bound on the Brane Flux Quanta

In this appendix we give a rough estimate for the lower bound on n_+ as motivated in section 3.3. To this end we write (3.17) as

$$\frac{V(\mathcal{V})}{V_0} = \frac{1}{\mathcal{V}^3} f(\mathcal{V}) \quad (\text{C.1})$$

where $f(\mathcal{V}) = A - B \log^{3/2}(C\mathcal{V}) + D\mathcal{V}^{1/3}$. Furthermore, we choose to expand the potential at the minimum $\mathcal{V}_{\min.}$ as

$$\frac{V(\mathcal{V})}{V_0} = \frac{f''(\mathcal{V}_{\min.})}{2\mathcal{V}^3} (\mathcal{V} - \mathcal{V}_{\min.})^2 + \dots \quad (\text{C.2})$$

Maximising (C.2) gives¹⁴ $\mathcal{V}_{\max.} = 3\mathcal{V}_{\min.}$ and

$$\frac{V(\mathcal{V}_{\max.})}{V_0} \simeq \frac{2}{3^3} \frac{f''(\mathcal{V}_{\min.})}{\mathcal{V}_{\min.}}. \quad (\text{C.3})$$

¹⁴Actually, as $(\mathcal{V}_{\max.} - \mathcal{V}_{\min.}) > \mathcal{V}_{\min.}$ the expansion (C.2) breaks down. However, the calculation still gives a first idea for the required size of n_+ which is confirmed numerically in figure 3.

Computing $f''(\mathcal{V}_{\min.})$, keeping only the leading term in $\log(C\mathcal{V}_{\min.})$, and using $f'(\mathcal{V}_{\min.}) = 0$ one finds

$$\frac{V(\mathcal{V}_{\max.})}{V_0} \simeq \frac{2}{3^5} \frac{D}{\mathcal{V}_{\min.}^{8/3}}. \quad (\text{C.4})$$

An estimate of how big the uplift can be such that it does not destroy the local minimum of the potential is given by requiring

$$\frac{V(\mathcal{V}_{\max.})}{V_0} \gtrsim \frac{\delta V(\mathcal{V}_{\min.})}{V_0} = \frac{\delta D}{\mathcal{V}_{\min.}^{8/3}}. \quad (\text{C.5})$$

This then implies

$$\frac{\delta D}{D} = \frac{\delta n}{n_+} \simeq \frac{1}{2} \left(\frac{n_-}{n_+} \right)^2 \lesssim \frac{2}{3^5} \simeq 0.008. \quad (\text{C.6})$$

Thus, $n_+ \geq 8$, $n_- = 1$ is ok. Obviously, this is only a very coarse analysis. However, figure 3 shows a plot of the potential with the numbers derived in section 3.3 and for $n_+ = 12$, $n_- \in \{0, 1\}$. This confirms our analysis.

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